Photon Noise Generation of Cathode-Ray Tube Display Systems

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The mean square fluctuation components associated with the various luminous fluxes of ambient and generated light emitted from a cathode-ray tube (CRT) system are derived and combined to represent the photon noise component. A rationale for the criteria required for the discernment of noisy shades of gray as displayed by CRT systems is developed. The criteria are applied to an actual CRT system and verified with experimental results.

I. Introduction

The importance of determining photon noise relates to the evaluation of the photometric quality and discernible limitation of the reproduced image displayed on a cathode-ray tube (CRT) system. Fluctuations in the stream of photons, commonly known as photon noise, are generated by various phenomena occurring in the light dynamics of the CRT system. The fluctuations of the light emitted from a typical CRT system are divided into two parts in this article. The first part describes the fluctuations related to the ambient light reflected from the system. The second part describes the fluctuations of the light which is generated in the CRT. These fluctuations are then combined to represent the total mean fluctuation of the total light emitted from the system. Calculated luminance levels required for the discernment of noisy gray shades generated by a selected CRT system are also shown and compared to experimental results.

II. Photon Noise Derivations

Light emitted from a CRT system (Fig. 1) is comprised of external light reflected from the CRT system, and light generated by the impingement of the electron beam upon the cathodoluminescent material. The fluctuations associated with these two light sources are derived and combined in the subsequent sections.

A. Ambient Light

The energy E of a photon is the product hf, where h is Planck's constant and f is the frequency. Consider a monochromatic flux of photons $n(\lambda)$ with wavelength λ flowing through an area dA. The energy of these photons may be expressed photometrically as

$$\frac{dE}{dA} = dL = K_v ch\, n(\lambda)\, V(\lambda) \frac{d\lambda}{\lambda} \qquad (1)$$

where K_v is a factor to convert energy to lumens, c is the velocity of light, and $V(\lambda)$ is the visual response. Integrating Eq. (1) over an incremental wavelength $\Delta\lambda$ at a wavelength λ results in

$$L = K_v hc\, n(\lambda)\, V(\lambda) \int_{\lambda}^{\lambda+\Delta\lambda} \frac{d\lambda}{\lambda} \qquad (2)$$

or

$$L = K_v hc\, n(\lambda)\, V(\lambda) \frac{\Delta\lambda}{\lambda} \qquad (3)$$

where L is the luminous flux at wavelength λ. The radiant flux of photons $n(\lambda)$ may be written from Eq. (3) as

$$n(\lambda) = \frac{L\lambda}{K_1 V(\lambda)\, \Delta\lambda} \qquad (4)$$

where $K_1 = K_v hc$.

It also follows that the average radiant flux of photons $\overline{n(\lambda)}$ at wavelength λ required to produce a mean luminous flux $\overline{L}$ is

$$\overline{n(\lambda)} = \frac{\overline{L}\lambda}{K_1 V(\lambda)\, \Delta\lambda} \qquad (5)$$

1. Noise from the filter screen. A monochromatic radiant flux of photons $n_a(\lambda)$ external to the CRT system enters at the filter screen having a transmissivity coefficient $\tau_f(\lambda)$. Of the amount which does not pass on to the faceplate, a portion is absorbed $\alpha_f(\lambda)$ by the filter screen and the remaining is reflected from the screen by the amount $\rho_f(\lambda)$ (Ref. 1). The reflectance factor $\rho_f(\lambda)$ may be interpreted to represent the probability of the incident photons not dissipated, whereas the factor $\alpha_f(\lambda) = 1 - \rho_f(\lambda)$ represents the probability of the incident photons dissipated. These probabilities are valid provided that the individual events of the photons are completely random.

Because the considered beam of light is monochromatic, the values of all parameters will be understood to be a function of wavelength and will not be expressed as such unless noted otherwise. This is made to simplify expressions and thereby provide ease of reading.

From the theory of partition noise (Ref. 2), the mean square fluctuation in the radiant flux of photons

$$\rho_f (1 - \tau_f)\, n_a$$

reflected from the filter screen may be expressed as

$$\overline{(n_a - \overline{n}_a)_f^2} = \rho_f (1 - \rho_f)(1 - \tau_f)\, n_a = F_f^2 \cdot n_a \qquad (6)$$

where

$$F_f^2 = \rho_f (1 - \rho_f)(1 - \tau_f) \qquad (7)$$

2. Noise generated by the glass faceplate. The radiant flux of photons impinging upon the outer surface of the glass faceplate is $\tau_f n_a$. By nature of the randomness of the reflection and absorption processes which these photons encounter, partition noise is generated. The mean square noise component associated with the flux of photons reflected from and those entering the faceplate can be written as

$$\overline{(n_a - \overline{n}_a)^2} = k_p\, \tau_f\, n_a \qquad (8)$$

where

$$k_p = \rho_g (1 - \rho_g) \qquad (9)$$

and ρ_g is the reflectivity coefficient of the glass faceplate. The mean square noise component associated with the flux of photons $\rho_g \tau_f^2 n_a$ reflected from and finally passing through the filter screen can be expressed as

$$\overline{(n_a - \overline{n}_a)_{g1}^2} = \tau_f^3\, k_p\, n_a \qquad (10)$$

Equation (10) is the first noise component term of the light reflections from the glass faceplate. The flux of photons $(1 - \rho_g)\, \tau_f n_a$ entering the faceplate proceed to be reflected from the inner surface of the faceplate and finally emerge out of the CRT system carrying the second noise component term. The flux impinging upon the inner surface is $\tau_g (1 - \rho_g)\, \tau_f n_a$ of which $\rho_g \tau_g (1 - \rho_g)\, \tau_f n_a$ is reflected and $\tau_g (1 - \rho_g)^2\, \tau_f n_a$ passes to the cathodoluminescent material (Ref. 3). This material is commonly called the phosphor screen (Ref. 4) and will be denoted as such here.

As the light passes through the glass faceplate, the portion absorbed by the faceplate is proportional to $(1 - \tau_g)$. The transmission process, therefore, generates partition noise to the flux of photons passing through. Assuming that τ_g is not a function of faceplate thickness, the average flux of photons in the considered stream passing through the faceplate is $[(1+\tau_g)/2]\,(1-\rho_g)\,\tau_f n_a$.

The mean square noise component generated during the transmission process is

$$\tau_g(1-\tau_g)\frac{(1+\tau_g)}{2}(1-\rho_g)\,\tau_f\,n_a$$

This quantity can be related to the flux of photons transmitted and expressed as

$$\overline{(n_a-\overline{n}_a)^2_{T1}} = k_T\,\tau_g(1-\rho_g)\,\tau_f\,n_a \tag{11}$$

where

$$k_T = \frac{(1-\tau_g)^2}{2} \tag{12}$$

The flux of photons entering the faceplate, however, contains a noise component given in Eq. (10). This component is contained in the flux being transmitted through the glass faceplate. The mean square components by Eqs. (10) and (11) can be algebraically added to give the total mean square noise component associated in the flux of photons impinging upon the inner surface of the faceplate as follows:

$$\overline{(n_a-\overline{n}_a)^2_{T2}} = \tau_g^2\tau_f^3 k_p n_a + k_T\,\tau_g(1-\rho_g)\,\tau_f\,n_a \tag{13}$$

Similarly, the mean square noise component associated with the flux reflected from the inner surface can be written as

$$\overline{(n_a-\overline{n}_a)^2_{T3}} = \rho_g^2\,\overline{(n_a-\overline{n}_a)^2_{T2}} + k_p\,\tau_g(1-\rho_g)\,\tau_f\,n_a \tag{14}$$

As these photons are transmitted through the faceplate until they impinge upon the outer surface for another series of re-reflections and transmissions, partition noises are again generated in the transmission and reflection processes. By applying the noise generation argument described for Eqs. (13) and (14), the flux of photons $(n_a)_{g2}$ emerging from the filter screen as the second reflection term is

$$(n_a)_{g2} = (1-\rho_g)\,k_p\,\tau_g^2\tau_f^2\,n_a \tag{15}$$

and the mean square fluctuation associated in the flux $(n_a)_{g2}$ can be shown to be

$$\overline{(n_a-\overline{n}_a)^2_{g2}} = \tau_f^3\tau_g^2\left\{k_p^2 + (1-\rho_g)^2 k_T k_p + \frac{(1-\rho_g)^2}{n_a}\overline{(n_a-\overline{n}_a)^2_{T3}}\right\} n_a \tag{16}$$

Equation (16) is the second noise component term of the light reflections. Since there are an infinite number of subsequent reflections and mean square fluctuation components which can be continually described, the total mean square fluctuation component $\overline{(n_a-\overline{n}_a)^2_g}$ associated with the cumulative reflective fluxes is

$$\overline{(n_a-\overline{n}_a)^2_g} = \overline{(n_a-\overline{n}_a)^2_{g1}} + \overline{(n_a-\overline{n}_a)^2_{g2}} + \cdots \overline{(n_a-\overline{n}_a)^2_{gn}} \tag{17}$$

Combining this series expression, and eliminating the products having second order effects, Eq. (17) reduces to

$$\begin{aligned}\overline{(n_a-\overline{n}_a)^2_g} &\cong \tau_g^2\tau_f^3\,\{k_p^2 + (1-\rho_g)^2\,[k_T k_p \\ &\quad + k_p\,\tau_g(1+\tau_g-\rho_g) + k_T\,\tau_g(1-\rho_g)]\}\cdot n_a \\ &= F_p^2\cdot n_a\end{aligned} \tag{18}$$

The inter-reflections of the radiant fluxes inpinging upon the filter screen and reflecting back to the outer surface of the glass faceplate for similar light dynamics described above were surveyed. Each reflected flux of photons would be reduced by the factor $\rho_f\rho_g(1-\tau_f)$. Substituting values for the parameters of practical CRT systems, the resulting reflection terms would have less than a second-order effect upon the value of the total mean square fluctuation component generated in the system. These inter-reflections are considered negligible and therefore are not included.

3. Noise from the phosphor screen. The series of radiant fluxes emerging from the inner surface of the glass faceplate toward the phosphor screen including their respective mean square fluctuation components can be combined and expressed as a single radiant flux of photons including a mean square fluctuation component associated with it. This flux then impinges upon the phosphor screen and generates partition noise due to the random distribution occurring in the reflection and absorption processes. The reflective flux proceeds through the glass faceplate and filter screen, during which further partition noises are generated during transmission and reflection processes. The subsequent reflection terms were neglected because their values contribute less than third-order effects to the total mean square fluctuation component. The first term only was, therefore, considered.

The total flux impinging upon the phosphor screen from the faceplate can be shown to be (Ref. 3)

$$(n_a)_i = \frac{(1-\rho_g)^2\,\tau_g\,\tau_f}{(1-\rho_g^2\,\tau_g^2)}\cdot n_a \tag{19}$$

The mean square fluctuation contained in $(n_a)_i$ in its reduced expression is

$$\overline{(n_a - \bar{n}_a)_i^2} \cong (1 - \rho_g)\,\tau_g\,\tau_f\,\{k_p\,[1 + \tau_g(1 - \rho_g)] + k_T(1 - \rho_g)^2\} \cdot n_a \qquad (20)$$

The flux of photons emerging from the CRT system through the filter screen because of $(n_a)_i$ can be shown to be (Ref. 3)

$$(n_a)_s = \frac{(1 - \rho_g)^3\,\tau_g^2\,k_s\,\tau_f}{(1 - \rho_g^2\,\tau_g^2)} \cdot n_a \qquad (21)$$

where $k_s = \rho_s(1 - \rho_s)$, and ρ_s is the reflectivity coefficient of the phosphor screen. The mean square fluctuation component in its reduced form associated with (n_a) is

$$\overline{(n_a - \bar{n}_a)_s^2} \cong \tau_f^3\,\tau_g^2\,(1 - \rho_g)^2\,[J_1 + (1 - \rho_g)^2\,\rho_s\,k_s\,\tau_g\,J_2] \cdot n_a = F_s^2 \cdot n_a \qquad (22)$$

where

$$J_1 = k_p\,k_s + (1 - \rho_g)^2\,k_T\,k_s + (1 - \rho_g)^2\,\tau_g\,k_p\,\rho_s + (1 - \rho_g)^4\,\tau_g\,k_s$$

$$J_2 = k_p\,[1 + \tau_g(1 - \rho_g)] + k_T(1 - \rho_g)^2$$

4. Combining the fluctuations. The total flux of photons $(n_a)_t$ emerging from the CRT system due to the reflections of the incident flux of ambient light n_a is given as (Ref. 3)

$$(n_a)_t = \tau_f^2 \left\{\rho_g + \frac{(1 - \rho_g)^2\,\tau_g^2\,H_a}{(1 - \rho_g\,\tau_g^2\,H_a)}\right\} \cdot n_a \qquad (23)$$

where

$$H_a = \rho_g + \frac{\rho_s(1 - \rho_g)^2}{1 - \rho_g\,\rho_s} \qquad (24)$$

The mean square fluctuation component $\overline{(n_a - \bar{n}_a)_t^2}$ associated with the total radiant flux output $(n_a)_t$ is found by combining the results of Eqs. (6), (18), and (22) as follows:

$$\overline{(n_a - \bar{n}_a)_t^2} = (F_f^2 + F_p^2 + F_s^2) \cdot n_a \qquad (25)$$

B. Generated Light

The phosphor screen, comprised of cathodoluminescent material, will luminesce when impacted by electrons. The light produced at the emitting surface of the phosphor screen will fluctuate and is characterized by the quality of the electron beam and the properties of the cathodoluminescent material. The three prime sources which can cause the fluctuation to occur are the residual noises in the electron beam, the process of photon conversion, and the granularity of the phosphor screen.

1. Noise in the electron beam. The electron beam current will fluctuate because of noise generated in the CRT circuit. Initially, the signal entering the CRT circuit may contain a mean square fluctuation voltage $\overline{(v_{in} - \bar{v}_{in})^2}$ produced through the receiver system, transmitter power, etc. This fluctuation is usually low (Ref. 4).

The fluctuation generated in the CRT amplifier is characterized by thermal agitation caused by the random motion of the free electrons in the resistor in the input stage of the amplifier (Refs. 4 and 5). The mean square fluctuation voltage can be expressed as

$$\overline{(v - \bar{v})^2} = 4KT \int_{f_1}^{f_2} R(f)\,A(f)\,df \qquad (26)$$

where K is Boltzmann's constant, T is the absolute temperature of the resistor, $R(f)$ is the equivalent noise resistance of the system impedance, and $A(f)$ is the power amplification of the system. The values of $R(f)$ and $A(f)$ may be a function of the frequency f within the bandwidth $f_2 - f_1$.

The electron beam also contains shot noise developed by the emission anode current i of the beam. The mean square fluctuation current is given by

$$\overline{(i - \bar{i})^2} = 2e\,(f_2 - f_1)\,i \qquad (27)$$

where e is the charge of an electron, and i is the dc anode current. There are other noise components in the electron beam current which occur from flicker effects, space-charge limitations, transit-time effects and secondary emission. However, these noise components are low relative to the value developed by shot noise in typical well-designed CRT systems operating at common frequency levels (Refs. 4 and 6). These noise components can, therefore, be neglected.

The total mean square fluctuation current in the electron beam can now be written from Eqs. (26) and (27) as

$$\overline{(i - \bar{i})_t^2} = \int_{f_1}^{f_2} \overline{(v_{in} - \bar{v}_{in})^2}\,\frac{A(f)}{R_i^2(f)}\,df + 4KT \int_{f_1}^{f_2} F(f)\,R(f)\,df + \overline{(i - \bar{i})^2} \qquad (28)$$

where $F(f)$ is the noise figure of the amplifier system.

2. ***Photon conversion.*** The energy of the electron beam upon impact with the phosphor screen is converted to quencher center recombinations, backscattering of incident electrons, long-wavelength radiation from the electron, production of secondary electrons, thermal heat transfer to environs, and finally the emission of the generated photons (Ref. 7).

Let $\bar{n}_e$ be the average arrival of electrons per second at the phosphor screen, and cover an area dA visible at the light emitting surface of the screen. The average radiant flux of generated photons $\bar{n}_g$ emitted as a function of wavelength can then be written as

$$\overline{n_g(\lambda)} = \overline{\eta(\lambda, i)} \cdot \bar{n}_e \tag{29}$$

where $\overline{\eta(\lambda, i)}$ is the mean electron–photon conversion function of the given cathodoluminescent material and may be a function of λ and beam current i. Since $\overline{\eta(\lambda, i)}$ represents the fraction of electron energy converted to light and $(1 - \overline{\eta(\lambda, i)})$ represents the fraction transferred to other mechanisms, it follows that partition noise is developed in the emitted radiant flux of photons n_g and expressed as

$$\overline{[n_g(\lambda) - \overline{n_g(\lambda)}]^2} = [\overline{\eta(\lambda, i)}]^3 [1 - \overline{\eta(\lambda, i)}] \cdot \bar{n}_e \tag{30}$$

3. ***Granularity.*** The cathodoluminescent material is comprised largely of inorganic crystalline particles of sizes varying in accordance with their size distribution. These particles are usually deposited on the CRT glass faceplate either by a settling process or by electrophoresis (Ref. 8). Settled films are not as homogeneous and appear to have higher graininess than do films processed by electrophoresis. The amplitude of randomness of cathodoluminescence was measured and shown to be greater for settled films because of the greater nonhomogenuity of particle size and deposition (Ref. 8). It follows that granularity, the objective measurement of the diffusion of light by the grains in the granular structure, must be considered as a noise generating source.

Let $\eta(\lambda, i)$ signify the electron–photon conversion function at an arbitrary position located on the faceplate. Equation (29) describes the mean photon radiant flux with wavelength λ generated for the phosphor screen. However, the photon radiant flux generated at the arbitrary position can similarly be written as

$$n_g(\lambda) = \eta(\lambda, i) \cdot \bar{n}_e \tag{31}$$

By squaring the difference between these two emissions, the mean square fluctuation component of the emitted photon flux is

$$\overline{[n_g(\lambda) - \overline{n_g(\lambda)}]^2} = [\eta(\lambda, i) - \overline{\eta(\lambda, i)}]^2 \cdot (\bar{n}_e)^2 \tag{32}$$

Let the granularity factor σ be defined as

$$\sigma(i) = \int_\lambda [\eta(\lambda, i) - \overline{\eta(\lambda, i)}]\, d\lambda \tag{33}$$

It follows that the mean square fluctuation component of the radiant flux over the spectrum generated is

$$\overline{(n_g - \bar{n}_g)^2} = \sigma^2(i) \cdot \bar{n}_e^2 \tag{34}$$

4. ***Combining the fluctuations.*** The current fluctuation of the electron beam (Eq. 28) directly converts to a photon fluctuation component. If the root mean square (rms) value of this fluctuation in the beam current were divided into the beam current, $i/[\overline{(i - \bar{i})_i^2}]^{1/2}$, the quotient is identified as the rms signal to noise ratio (SNR) of the beam current. Equation (28) can then be converted to a mean square fluctuation component of the photon radiant flux generated as follows:

$$\overline{[n_g(\lambda) - \overline{n_g(\lambda)}]^2} = \left[\frac{\overline{\eta(\lambda, i)}}{\mathrm{SNR}(i)}\right]^2 \left(\frac{i}{e}\right)^2 \tag{35}$$

where SNR (i) may be a function of the beam current.

Combining Eqs. (30), (34), and (35) yields the total mean square fluctuation of the total generated photon radiant flux emitted from the phosphor screen surface as follows:

$$\begin{aligned}\overline{[n_g(\lambda) - \overline{n_g(\lambda)}]_t^2} &= \left[\left\{\eta^3(\lambda, i)[1 - \eta(\lambda, i)] + \frac{i}{e}\left[\sigma^2(i) + \left(\frac{\eta(\lambda, i)}{\mathrm{SNR}(i)}\right)^2\right]\right\}\right] \cdot \bar{n}_e(\lambda) \\ &= N_g^2(\lambda, i) \cdot \bar{n}_e(\lambda)\end{aligned} \tag{36}$$

This noise component characterized by the factor $N_g^2(\lambda, i)$ passes through the CRT system in which partition noises due to reflection and transmission processes enhance the noise component of the generated beam. Following the same arguments described in *Section A*, it can be shown that the total mean square fluctuation component of the generated beam emitted from the filter screen of the CRT system is

$$[n_g(\lambda) - \overline{n_g(\lambda)}]^2_{\text{sum}} = \tau_f^2 \{\tau_g (1 - \rho_g) [(1 - \rho_g) \tau_g k_\rho + k_T (1 - \rho_g)^2 + k_\rho] + (1 - \rho_g)^2 \tau_g^2 N_g^2 (\lambda, i)\} \cdot \overline{n}_e (\lambda)$$
$$= F_g^2 (\lambda, i) \cdot \overline{n}_e (\lambda) \quad (37)$$

The total radiant flux of photons generated $(n_g(\lambda))_t$ by the electron beam and emitted from the CRT system including all the inter-reflections (Ref. 3) is given as

$$[n_g(\lambda)]_t = \frac{(1 - \rho_g) \tau_f \tau_g K_g}{Q} \cdot n_g(\lambda) \quad (38)$$

where

$$K_g = \frac{1 - \rho_g}{1 - \rho_s \rho_g} \quad (39)$$

$$Q = \frac{1 - \rho_s \rho_g - \rho_g^2 \tau_g^2 - \rho_s \rho_g \tau_g^2 + 2\rho_g^2 \rho_s \tau_g^2}{1 - \rho_s \rho_g} \quad (40)$$

III. Total Noise and Total Light Emitted

The mean square fluctuation component of the ambient photon flux can be converted to luminance by using the square of the difference of Eqs. (4) and (5). As an example, Eq. (25) can be rewritten as

$$\overline{(L - \overline{L})_a^2} = [F_f^2 + F_\rho^2 + F_s^2] \cdot \left(K_1 V(\lambda) \frac{\Delta\lambda}{\lambda}\right)^2 \cdot n_a \quad (41)$$

Similarly Eq. (37) can be converted to luminance. The sum of this result with Eq. (41) will give the total mean square fluctuation component of the total light generated and reflected from the CRT system. This is written as

$$\overline{(L - \overline{L})_t^2} = \left(K_1 V(\lambda) \frac{\Delta\lambda}{\lambda}\right)^2 [(F_f^2 + F_\rho^2 + F_s^2) \cdot n_a + F_g^2 (\lambda, i) \cdot \overline{n}_e] \quad (42)$$

Since all the parameters expressed in Eq. (42) are functions of wavelength, the total root mean square (rms) component of the total luminance emitted over the visual spectrum is

$$\sqrt{\overline{(L - \overline{L})_t^2}} = \int_{\lambda_1}^{\lambda_2} [(F_f^2 + F_\rho^2 + F_s^2) \cdot n_a + F_g^2 (\lambda, i) \cdot \overline{n}_e]^{1/2} K_1 V(\lambda) \frac{\Delta\lambda}{\lambda} \quad (43)$$

The total radiant flux of photons emitted at wavelength λ out of the CRT system by both the ambient light and generated light is the sum of Eqs. (23) and (38). This flux can be converted to luminance by applying Eq. (4). Hence the total luminous flux L_t emitted from the CRT system over the visual spectrum is

$$L_t = K_1 \int_{\lambda_1}^{\lambda_2} V(\lambda) [(n_a(\lambda))_t + (n_g(\lambda))_t] \frac{d\lambda}{\lambda} \quad (44)$$

IV. Rationale for Discerning Gray Shades

To just discern the luminous flux of two adjacent gray-shade objects on a CRT screen, a certain minimum contrast between the displayed gray shades is required. This minimum contrast, called the threshold contrast factor C_T for a given spectrum, is a function of luminous flux of the object, the visual angle subtended, and the spatial frequency related to either the object domain or the retinal domain (Ref. 9). To just discern two adjacent equally sized gray-shade objects of uniform luminance levels L_1 and L_2 respectively viewed from a given distance, the minimum luminance difference is described by the relationship (Ref. 9)

$$L_1 - L_2 = C_T (L_1 + L_2) \quad (45)$$

However, when a fluctuation is associated with the luminous flux of each shade, the quality of the luminous flux is degraded and is said to be noisy. Let the fluctuation be described as the rms luminous noise component ΔL_0 of the object at luminance level L_0, and defined by the mean square fluctuation component as

$$\Delta L_0 = \sqrt{\overline{(L_0 - \overline{L}_0)^2}} \quad (46)$$

In the presence of noise, the fluctuating luminance levels of the two gray shades are such that the upper rms peak level of the lower luminance level L_2 will tend to blend in with the lower rms peak level of the higher luminance level L_1 as shown in Fig. 2. This blending results in lowering the probability of discernment of the gray shades, thereby not satisfying Eq. (45). When the luminance level of the lower peak of the upper shade $(L_1 - \Delta L_1)$ equals the luminance level of the upper peak of the lower shade $(L_2 + \Delta L_2)$, discernibility is not possible since the two gray shades in this condition at the interface blend together and appear as one shade. It is reasonable to conclude that the reliability of discernment can be increased to the threshold value if the difference of the luminance levels between the gray shades is increased. Therefore, to make certain that discernibility can be achieved, it is concluded that the described peak luminance levels be spread to satisfy Eq. (46) as follows:

$$(L_1 - \Delta L_1) - (L_2 + \Delta L_2) = C_T [L_1 - \Delta L_1 + L_2 + \Delta L_2] \tag{47}$$

The luminance difference ΔL_t required for the discernment of adjacent noisy gray shades is $L_1 - L_2$, the difference of the luminance levels described in Eq. (47). This luminance difference required for gray-shade discernment at a constant reliability was experimentally found for a given CRT system utilizing a P4 phosphor. These results are shown in Fig. 3 as a function of the brighter luminance level L_B of the gray shade (Ref. 10). The CRT configuration and operating conditions are given in Ref. 11. The measured rms SNR at white level was 110. The luminance output at the faceplate as a function of current density is given in Table 1 (Ref. 4).[1]

The luminance difference ΔL_t can also be determined from Eq. (47) by evaluating the noise components given in Eq. (47) by Eq. (43). The luminance level of each gray shade is described by Eq. (44). This computation was performed for a given CRT system for a luminance range extending over three orders of magnitude and based on the current densities given in Table 1 and the granularity factor of the phosphor screen to be 0.025.[2] The computation results are shown as a dashed curve in Fig. 3 and are in good agreement with the experimental results.

It can be seen in Fig. 3 that a progressive deviation of the computed result from the experimental result occurs at luminance levels above 171.3 cd/m^2 (50 ft-L). This deviation is due to the glaring characteristics of CRT screens which was not included in the computations. Figure 3 also includes a curve for a noiseless CRT system. The luminance difference required to discern noisy CRT gray shades is about four times greater than that required for noiseless gray shades.

V. Concluding Remarks

The luminance fluctuations generated in a CRT system are formulated and were verified by their application to an actual CRT system whose parameters and operating conditions were known.

[1]Also communication with Conrac Corp., CRT Dept.

[2]Communication with CBS Laboratories, Electron Physics Dept.

Acknowledgment

The author is grateful to Mr. Dean A. Howard for his interest and encouragement to make this study, and to Dr. Katsunori Shimada for his valuable help and contributions in the photon noise formulations.

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Table 1. Luminance output versus current density

Luminance output		Current density, A/cm^2
ft-L	cd/m^2	
140	479.6	0.2
115	394.0	0.1
80	274.1	0.06
60	205.6	0.04
32	109.6	0.02
0.32	1.1	0.0002

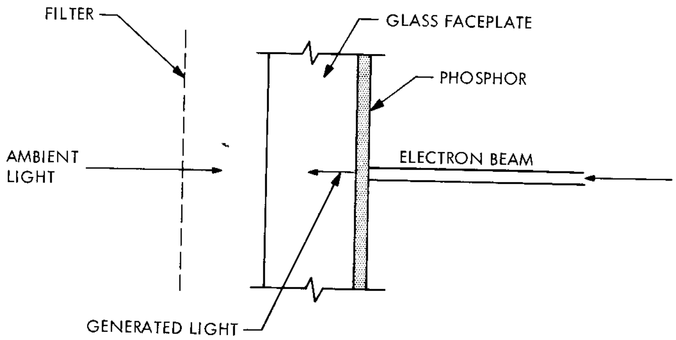

Fig. 1. Cathode-ray tube configuration

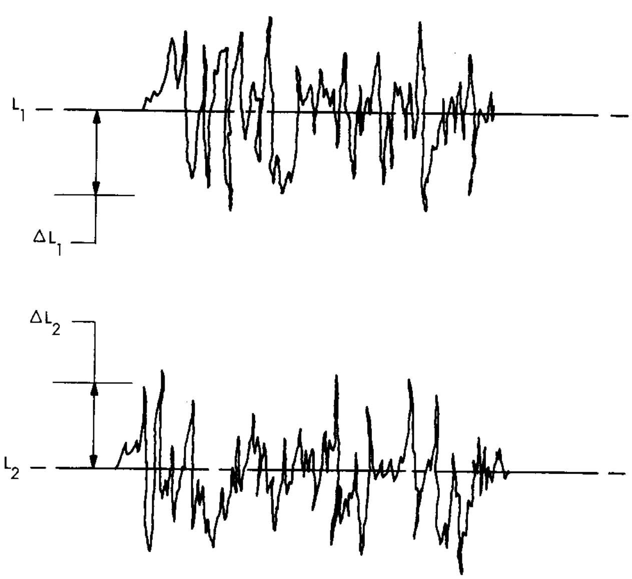

Fig. 2. Gray-shade luminous flux

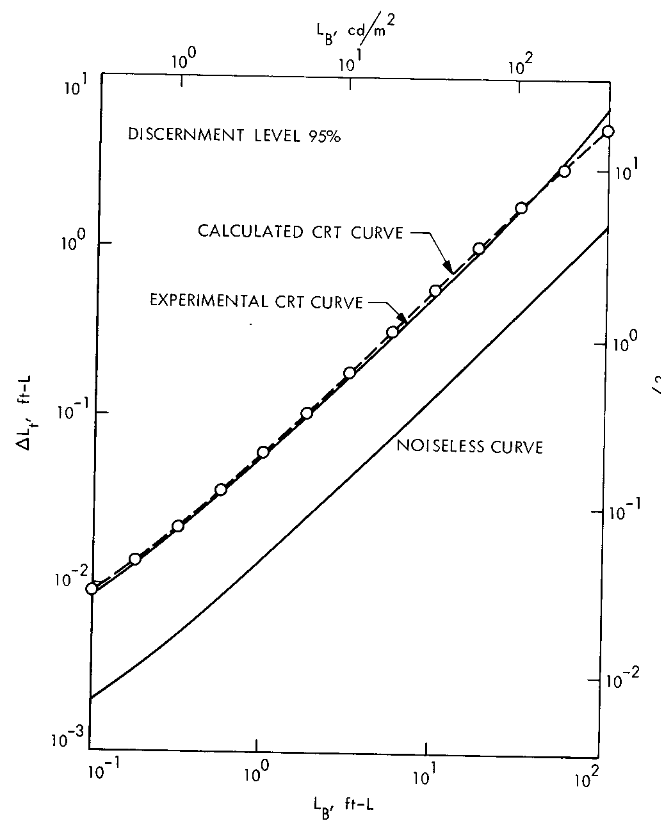

Fig. 3. Difference of luminance ΔL_t to discern gray shades vs luminance of brighter gray shade L_B